

ENGINEERING MECHANICS

STATICS

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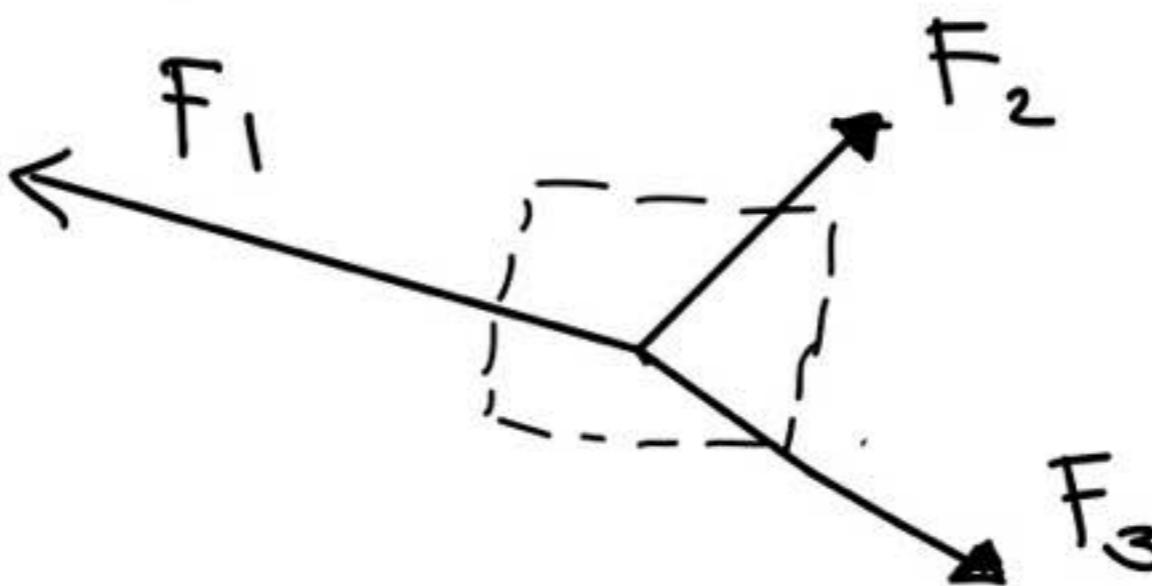
I. Basic Principles

1) Forces in equilibrium

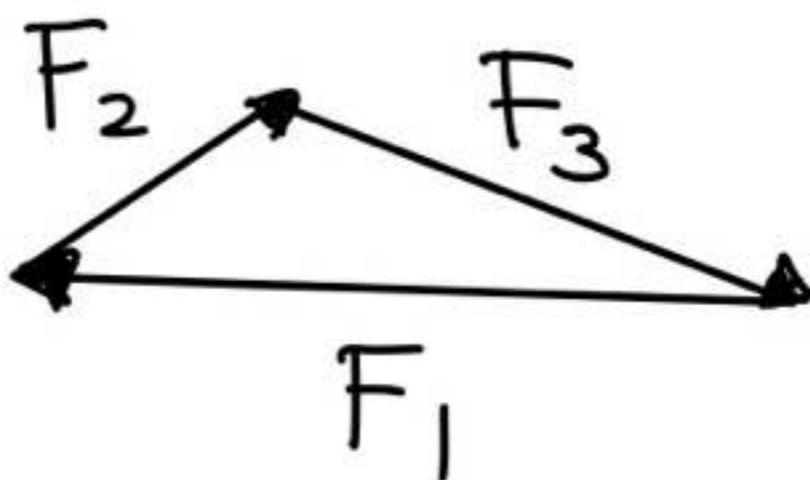
$$\sum F_x = 0$$

$$\sum F_y = 0$$

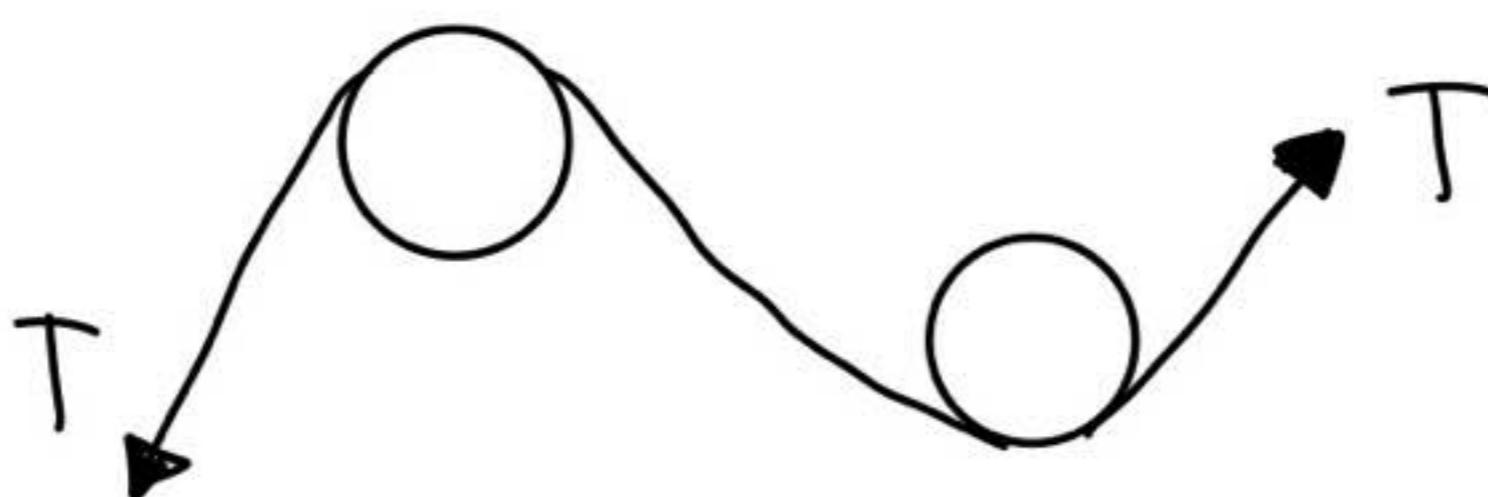
$$\sum M = 0$$



2) Forces in equilibrium form a closed polygon when connected tail of one to head of another.



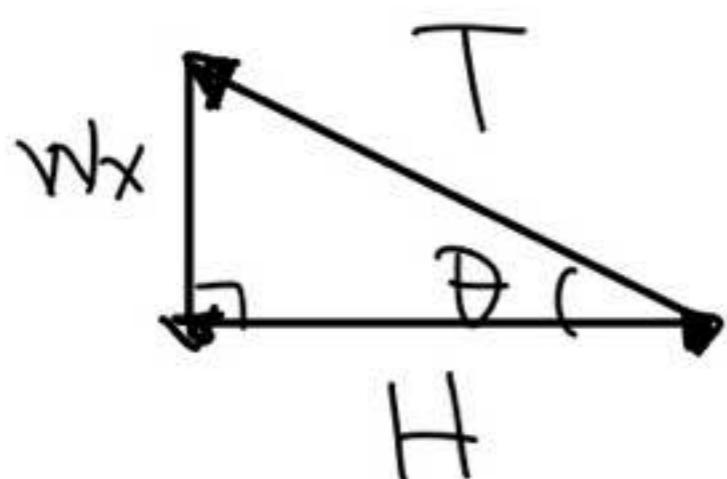
3) Tension on continuous cable running around a frictionless pulley is constant.



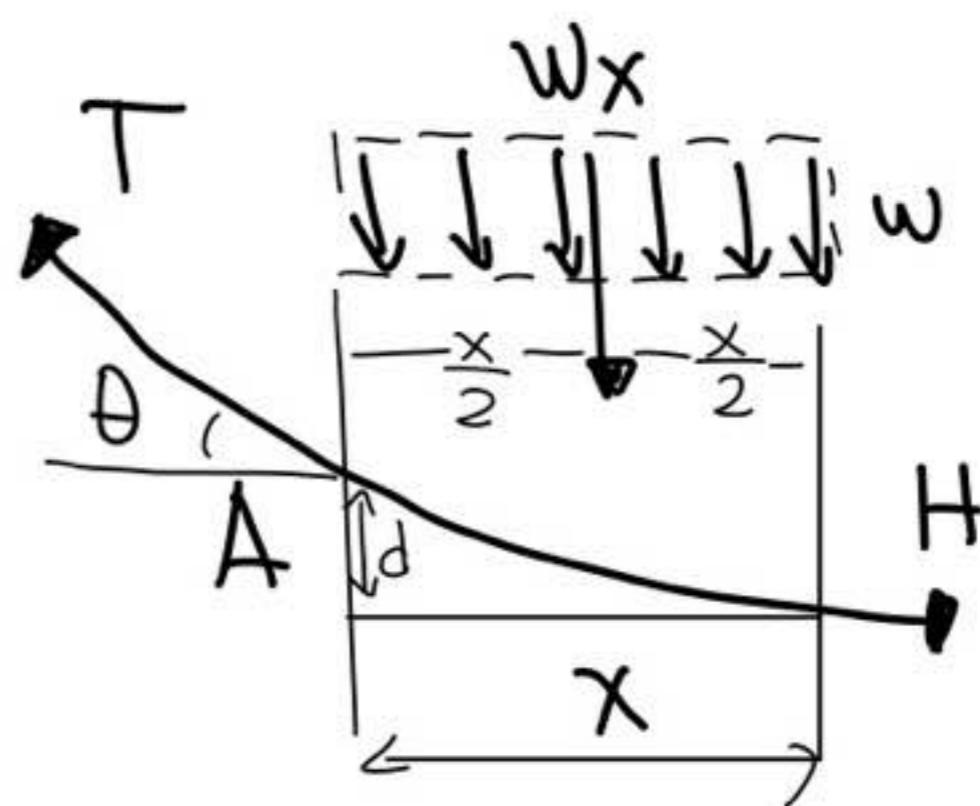
II. Parabolic Cables

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Note: the shape of a suspended cable is assumed to be a parabola when the uniformly distributed load is acting on the horizontal projection for half length :



force triangle



$$\sum M_A = 0$$

$$w x \left(\frac{x}{2} \right) - H d = 0$$

$$H = \frac{w x^2}{2d}$$

From force triangle:

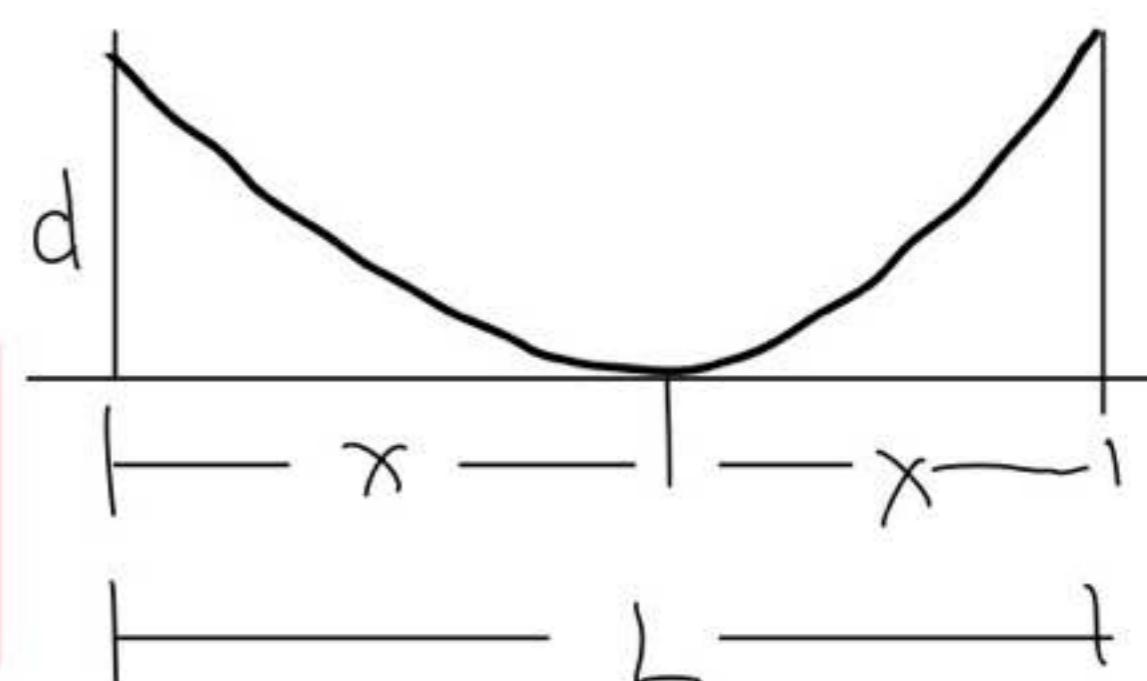
$$T = \sqrt{(w x)^2 + H^2}$$

$$\theta = \tan^{-1} \frac{w x}{H}$$

LENGTH OF CABLE :

Approximate:

$$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$$



For symmetrical cable:

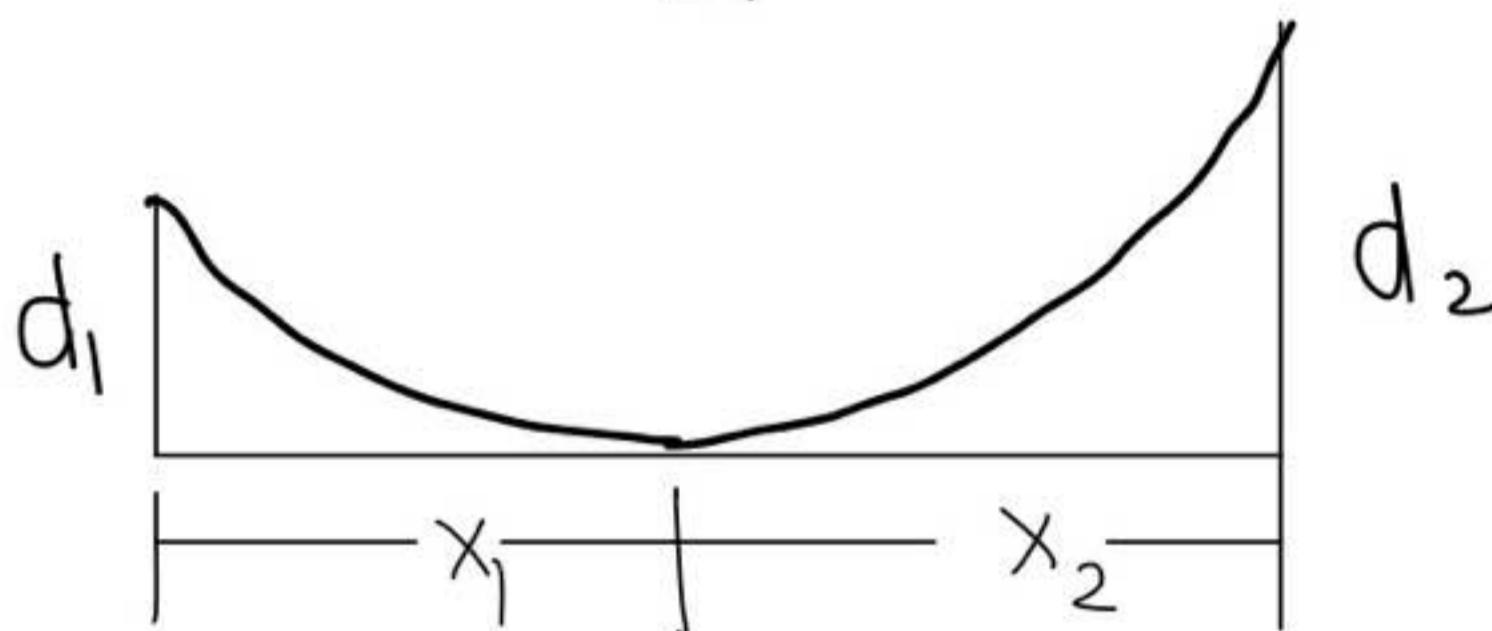
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$$L = 2x$$

For unsymmetrical cable

$$L_1 = 2x_1 \quad \& \quad L_2 = 2x_2$$

$$S_{\text{TOTAL}} = \frac{S_1 + S_2}{2}$$



note: for exact value, using integration using principle
on arclength.

$$dS = \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} \quad \text{or}$$

$$dS = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1} dy$$

III. Friction: f

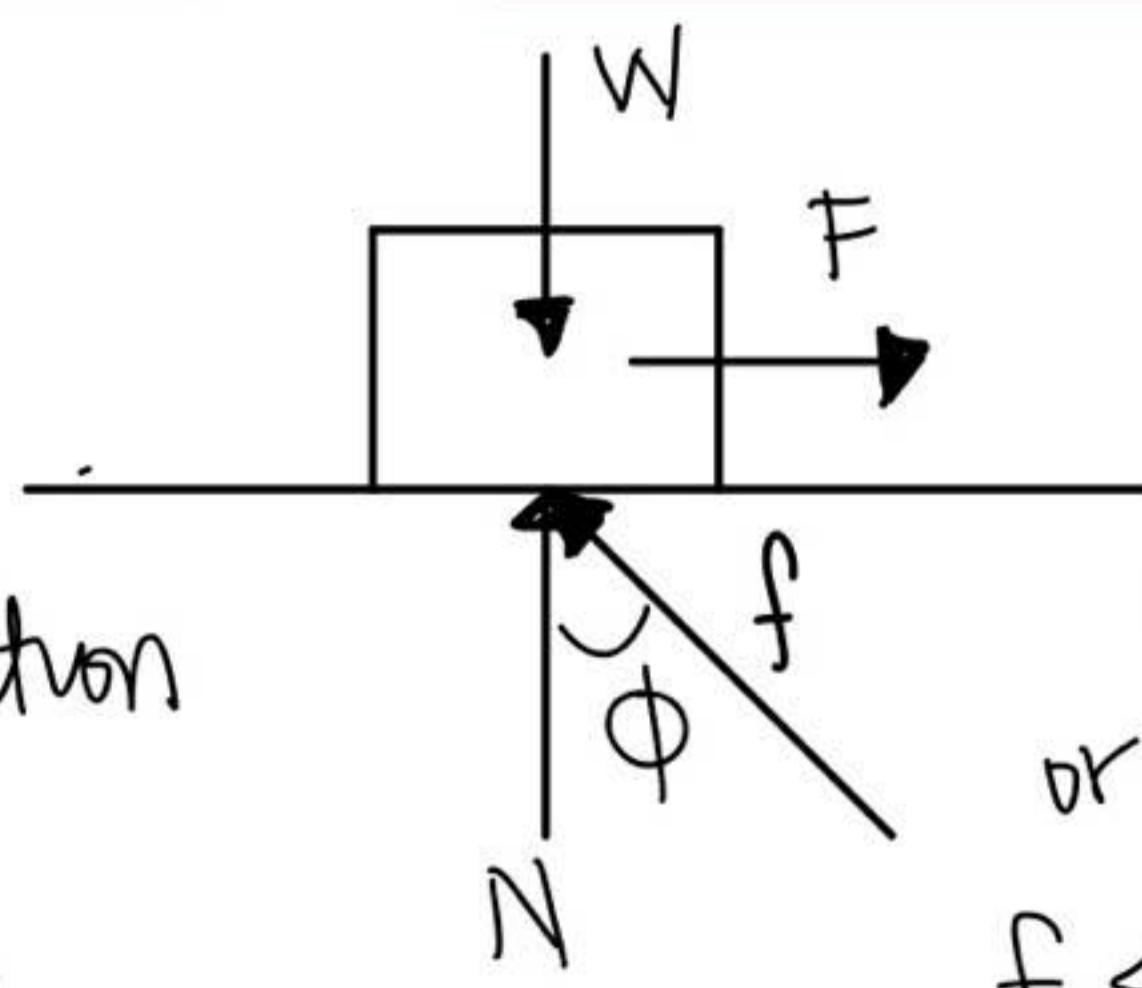
$$f \leq \mu N$$

N = normal reaction

μ = coefficient of friction

$$\mu = \tan \phi$$

ϕ = angle of friction



note:

$$f = \mu N, \text{ when}$$

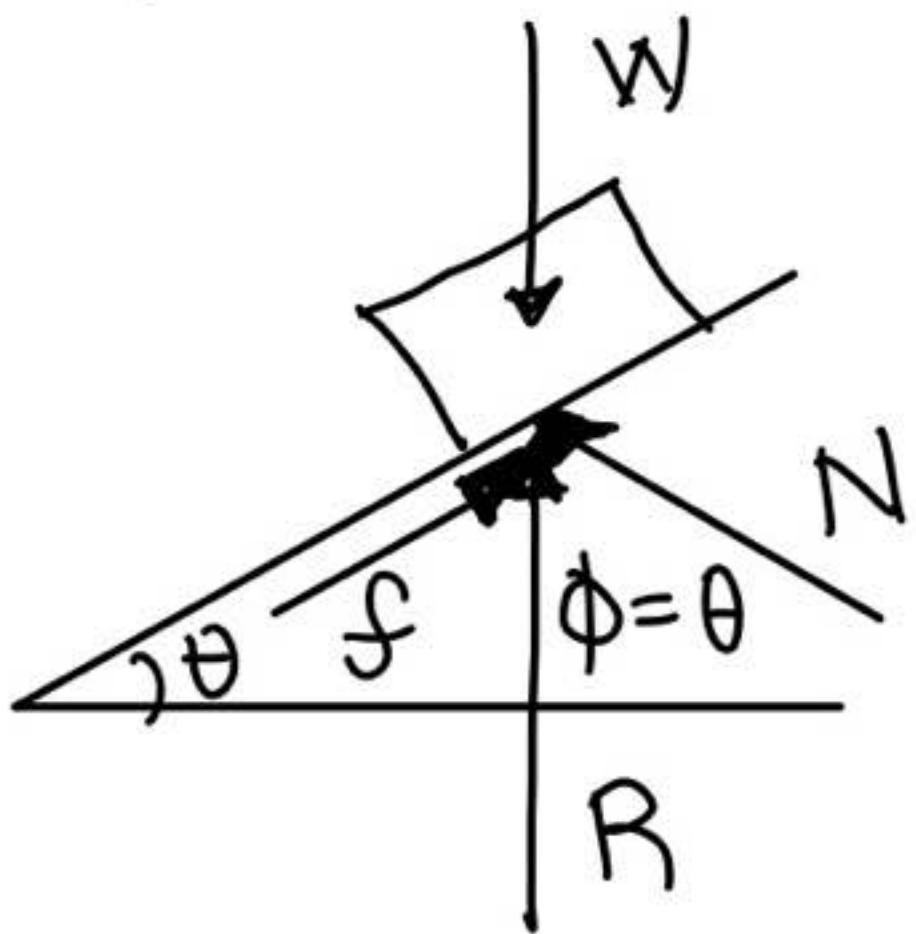
there is motion

or motion impends,
 $f < \mu N$ otherwise.

for maximum inclination of an inclined plane without sliding

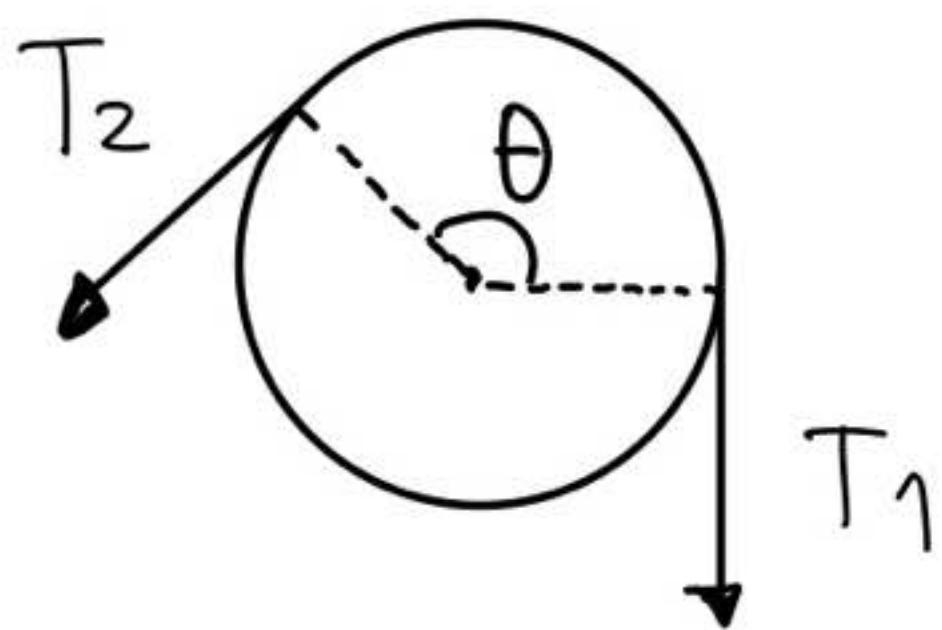
- to attain it, R & W must be collinear as the figure so that

$$\theta = \phi$$



IX Belt Friction

note: belt friction is present when the belt slide against the belt drum.



$$\frac{T_1}{T_2} = e^{\mu\theta}$$

μ = coefficient of belt friction
 θ = angle of contact (in radians) between the belt and the drum.

DYNAMICS

RECTILINEAR TRANSLATION

Case 1: Constant Velocity

$$S = Vt$$

S = distance travelled , m

V = average/constant velocity , m/s

t = time , s

Case 2: Constant acceleration

$$V_f = V_i + at$$

$$V_f^2 = V_i^2 + 2as$$

$$S = V_i t + \frac{1}{2}at^2$$

V_f = final velocity

V_i = initial velocity

a = acceleration , m/s²

note: a is negative for deceleration

Case 3: Constantly varying acceleration

$$V = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{\frac{dv}{ds}}{\frac{ds}{dt}} = \frac{v \frac{dv}{ds}}{s}$$

Freely Falling Bodies

- object moving vertically upward or downward acted only upon by the force of gravity
- for analysis, use the same formula above except

$$a = + 9.81 \text{ m/s}^2 \text{ when } V_i \text{ is downward or zero}$$

$$a = - 9.81 \text{ m/s}^2 \text{ when } V_i \text{ is upward}$$

s = vertical displacement from point 1 to point 2.

$s = (+)$ when in the same direction as V_i

$s = (-)$ when in the opposite direction as V_i

$V_f = (+)$ if in the same direction as V_i

Maximum height : $V = 0$

$$\therefore V_f^2 = V_i^2 + 2gs$$

$$0 = V_i^2 + 2(-g)s$$

$$\therefore s = \frac{V_i^2}{2g}$$

Time to reach maximum height :

$$V_f = V_i + at$$

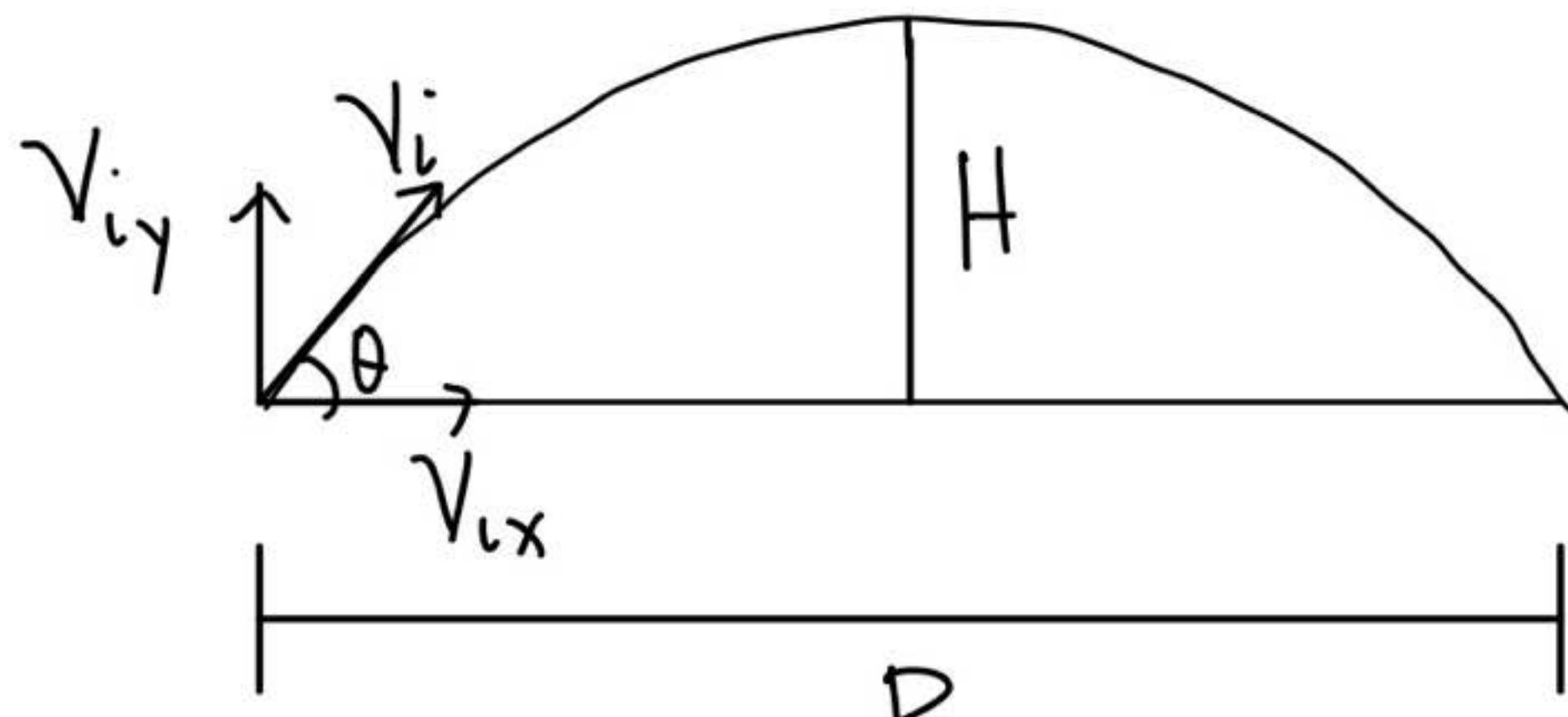
$$0 = V_i + (-g)t$$

$$t = \frac{V_i}{g}$$

total time of flight to return to the ground

$$t = \frac{2V_i}{g}$$

PROJECTILE



$$V_{ix} = V_i \cos \theta$$

V_i = initial velocity , m/s

H = maximum height , m

$$V_{iy} = V_i \sin \theta$$

R = range , m V_{ix} = horizontal , m/s
component of V_i

t = time , s V_{iy} = vertical
component of

From the equation of freely falling body , V_i , m/s

$$H = \frac{V_{iy}^2}{2g} = \frac{V_i^2 \sin^2 \theta}{2g}$$

Range, R

$$R = V_{ix} t$$

but $t = \frac{2V_{iy}}{g}$

$$R = V_i \cos \theta \frac{2V_i \sin \theta}{g}$$

$$R = \frac{V_i^2(2 \sin \theta \cos \theta)}{g} = \frac{V_i^2 \sin 2\theta}{g}$$

substitute: $V_i^2 = \frac{2gt}{\sin^2 \theta}$

$$R = \frac{\frac{2gt}{\sin^2 \theta} 2 \sin \theta \cos \theta}{g}$$

∴ $R = 4t \cot \theta$

ANGULAR MOTION

- use same formula as rectilinear motion except that you replace v by ω , s by θ and a by α .

θ = angle of rotation (rad)

ω = angular velocity (rad/s)

α = angular acceleration (rad/s^2)

Constant angular acceleration:

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$$\omega_f = \omega_i + \alpha t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\theta$$

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Note: $s = \theta R$

$$v = \omega R$$

$$a = \alpha R$$

CENTRIFUGAL FORCE

$$C.F. = \frac{W}{g} \cdot \frac{v^2}{R}$$

W = weight of moving object

v = tangential velocity (m/s)

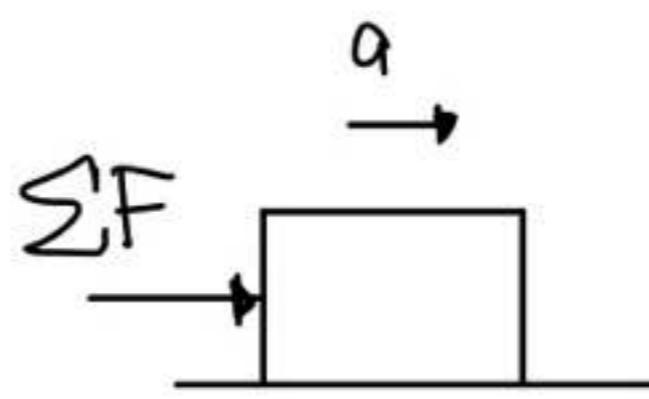
g = 9.81 m/s²

R = radius of curvature (m)

KINETICS

1) D'Alembert's Principle:

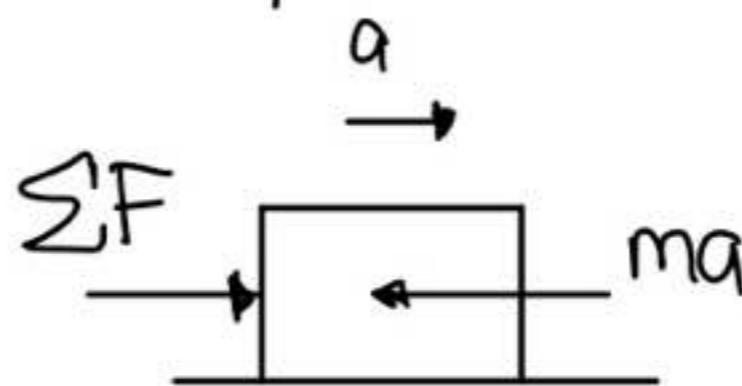
$$\sum F = ma$$



2) Newton's Method of Dynamic Equilibrium

$$\sum F = 0$$

→ includes inertia
force w/c is
equal to 'ma'



3) Work-energy Principle

$$\sum \left(\begin{array}{l} \text{Work done} \\ \text{on an object} \end{array} \right) = \Delta (\text{kinetic energy})$$

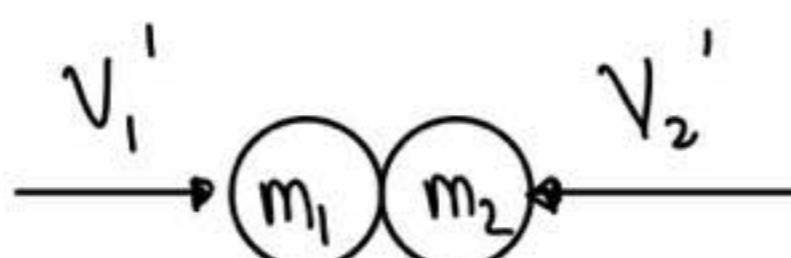
$$\sum F \cdot d = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\sum F \cdot d = \frac{1}{2} m (v_2^2 - v_1^2)$$

LAW OF CONSERVATION OF MOMENTUM



Before collision



After collision

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

$$e = \frac{v'_2 - v'_1}{v_2 - v_1}$$

note: $v = (+)$ to the right
 $(-)$ to the left
 $m = \text{mass in kg}$
 $e = \text{coefficient of restitution}$