EULER'S CRITICAL LOAD AND STRESS

For Hinged-Ended Columns:

Load

$$Pc = \frac{\pi^2 EI}{L^2}$$

Stress

$$Fa = \frac{\pi^2 E}{(L/r)^2}$$

For Fixed - Ended Columns:

Load

$$Pc = \frac{4\pi^2 EI}{L^2}$$

Stress

$$Fa = \frac{4\pi^2 E}{(L/r)^2}$$

where:

kL/r = max. effective slenderness ratio

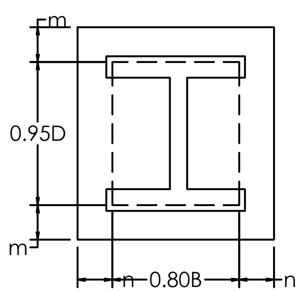
k = effective length factor

k = 1 for columns hinged at both sides

k = 0.50 fixed-fixed

k = 0.70 hinged-fixed

COLUMN BASE PLATE:



A. BEARING ON CONCRETE

Actual/Applied Bearing stress:

$$fp = \frac{P}{Ap}$$

where:

P = column load (kN)

Ap = contact surface between the base plate and conc. pedestal

Maximum Allowable compressive stress of conc.

On full area of a concrete support

$$Fp = 0.35\, fc'$$

On less than the full area of a concrete support

$$Fp = 0.35\, fc' \sqrt{A_2/A_1} \leq 0.70\, fc'$$

B. BENDING OF BASE PLATE

If m > n,

$$t = \sqrt{\frac{3\, fp\, m^2}{Fb}}$$

If n > m,

$$t = \sqrt{\frac{3\, fp\, n^2}{Fb}}$$

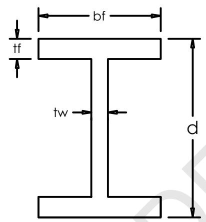
where:

fp = P/ A B

Fb = 0.75 Fy

SHEARING STRESS OF BEAMS

1. Maximum Web Shear Stress



$$fv = \frac{V_{max}}{d\, t_w}$$

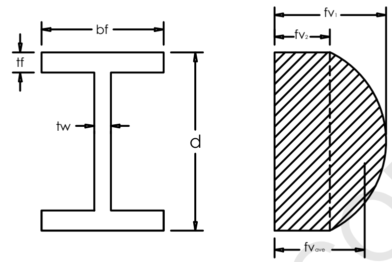
where:

V = max. shear force

d = depth of the beam

tw = web thickness

2. Average Shearing Stress in the Web



$$fv_1 = \frac{VQ_1}{I\, b}$$

$$fv_2 = \frac{VQ_2}{I\, b}$$

where:

$Q_1 = Q_f + Q_w$

$Q_2 = Q_f$

$$fv_{ave} = fv_2 + \frac{2}{3}(fv_1 - fv_2)$$

3. Maximum Vert./Hor. Shear Stress

$$fv_h = \frac{VQ}{I\, b}$$

where:

V = maximum shear of beam

Q = statical moment area

I = moment of inertia (mm^4)

b = base sheared

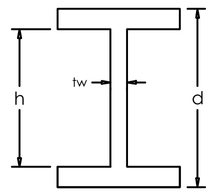
4. Shear flow

$$q = \frac{VQ}{I}$$

where:

q = shear flow (N/m)

5. Allowable Shear Stress



a. When $h/t_w < 998/\sqrt{Fy}$

Allowable shear stress

$$Fv = 0.40\, Fy$$

b. When $h/t_w > 998/\sqrt{Fy}$

Allowable shear stress

$$Fv = Fy\, Cv/2.89 \leq 0.40\, Fy$$

SPACING OF RIVETS OR BOLTS

$$S = \frac{R\, I}{V\, Q}$$

where:

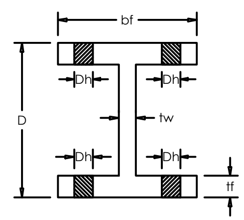
R = shear capacity of each bolts

V = maximum shear of beam

Q = statical moment area

MOMENT REDUCTION DUE TO THE PRESENCE OF HOLE IN BOTH FLANGE

- holes in beam generally will reduce its capacity. When the holes are located in the beam web, it reduces its shear capacity while holes in the beam flanges reduce its moment capacity.



A_{fn} = net flange area

A_{fg} = gross flange area

A_h = Ag - area of holes

Area of hole = $(D_h)(t_f)$

$D_h = d_b + 3$ mm

d_b = diameter of the bolt

NGCP SPECS:

1. Reduction of hole is neglected

When $0.50\, F_u A_{fn} > 0.60\, Fy\, A_{fg}$

2. Reduction of holes must be considered

When $0.50\, F_u A_{fn} < 0.60\, Fy\, A_{fg}$

Effective tension flange section:

$$A_{fe} = \frac{5\, F_u A_{fn}}{6\, F_y}$$

$A_{fn} = A_{fg}$ - area of holes

C. BENDING/FLEXURAL MEMBERS

Actual/Applied bending stress:

$$\boxed{fb = \frac{Mc}{I} = \frac{M}{S}}$$

ALLOWABLE STRESSES:

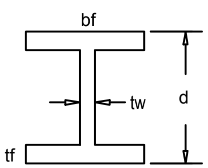
A. LATERALLY SUPPORTED BEAMS:

1. Compact Sections

$$\boxed{Fb = 0.66\ Fy}$$

Flange width - thickness ratio

$$\boxed{\frac{bf}{2tf} \leq \frac{170}{\sqrt{Fy}}}$$



Web depth - thickness ratio

$$\boxed{\frac{d}{tw} \leq \frac{1680}{\sqrt{Fy}}}$$

2. Non-compact Sections

$$\boxed{Fb = 0.6\ Fy} \quad \text{when} \quad \boxed{\frac{bf}{2tf} > \frac{170}{\sqrt{Fy}}}$$

3. Partially compact Sections

$$\boxed{Fb = Fy\left(0.79 - 0.00076\ \frac{bf}{2tf}\sqrt{Fy}\right)}$$

Flange width - thickness ratio

$$\boxed{\frac{bf}{2tf} > \frac{170}{\sqrt{Fy}}}$$

Web depth - thickness ratio

$$\boxed{\frac{bf}{2tf} > \frac{250}{\sqrt{Fy}}}$$

4. When Lb < Lc

$$\boxed{Fb = 0.66\ Fy}$$

$$Lc = \frac{200\ bf}{\sqrt{Fy}}$$

Lb = unbraced length of compression flange

5. When Lb > Lu

$$\boxed{Fb = 0.60\ Fy}$$

$$Lu = \frac{137{,}900\ Af}{Fy\ d}$$

6. When Lb > Lc and Lb < Lu

$$\boxed{Fb = Fy\left(0.79 - 0.00076\ \frac{bf}{2tf}\sqrt{Fy}\right)}$$

B. LATERALLY UNSUPPORTED BEAMS:

1. When Lb > Lc and Lb > Lu

$$\sqrt{\frac{703000C_b}{Fy}} \leq \frac{Lb}{r_t} \leq \sqrt{\frac{3520000\ C_b}{Fy}}$$

$$Fb = \left[\frac{2}{3} - \frac{Fy\,(Lb/r_t^2)}{10.55 \times 10^6\ Cb}\right] Fy$$

$$Fb = \frac{83 \times 10^3\ Cb}{Lb(d/Af)}$$

Use biggest value of Fb but should be $\boxed{\leq 0.60\ Fy}$

2. When Lb > Lc and Lb > Lu

$$\sqrt{\frac{703000C_b}{Fy}} \leq \frac{Lb}{r_t} > \sqrt{\frac{3520000\ C_b}{Fy}}$$

$$Fb = \frac{1170 \times 10^3\ Cb}{(Lb/rt)^2}$$

$$Fb = \frac{83 \times 10^3\ Cb}{Lb(d/Af)}$$

use bigger value of Fb but should be $\boxed{\leq 0.60\ Fy}$

where:

rt = radius of gyration of a section comprising the compression flange plus 1/3 of the compression web about the vertical axis.

$Cb = 1.75 + 1.05\,(M_1/M_2) + 0.30\,(M_1/M_2)^2$

$Cb \leq 2.3$

M_1 = smaller end moment

M_2 = bigger end moment

M_1/M_2 = negative (-) for single curvature

M_1/M_2 = positive (+) for double curvature

BENDING IN BOTH AXIS

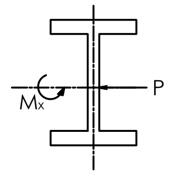
Beams Bending in Both Axis (Unsymmetrical Bending)

1. BENDING STRESS

a. If lateral loads pass thru the centroid of the beam section

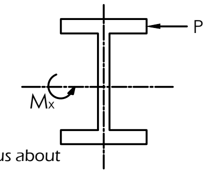
$$f_b = \frac{M_x C_x}{I_x} \pm \frac{M_y C_y}{I_y}$$

$$f_b = \frac{M_x}{S_x} \pm \frac{M_y}{S_y}$$



b. If lateral loads applied at the top flange and does not passes thru the centroid of the beam section

$$f_b = \frac{M_x}{S_x} \pm \frac{M_y}{S_y/2}$$



note:
Only one half of the section modulus about the y-axis is considered

2. SHEARING STRESS

$$f_v = \frac{V_x Q_x}{I_x b} \pm \frac{V_y Q_y}{I_y b}$$

3. USING INTERACTION EXPRESSION

a. $$\frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

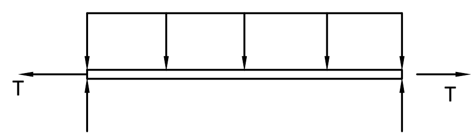
b. For compact laterally supported shapes:

$$\frac{f_{bx}}{0.66F_y} + \frac{f_{by}}{0.60\ F_y} \leq 1.0$$

For doubly symmetrical I and H shape members with compact flanges continuously connected to the web and bent about their weak axis, the allowable bending stress is 0.75 Fy.

Note: Consideration should be given to the question of lateral support for the compression flange which will indicate wether compact or non-compact sections.

TENSION WITH BENDING



$$f = \frac{T}{A} \pm \frac{M\,C}{I}$$

Members subject to both axial tension and bending shall be proportioned at all points along their length to satisfy the following equation:

1. BENDING IN ONE AXIS ONLY

$$\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} \leq 1.0$$

where:

fa = computed axial stress
fa = T/A
fb = computed bending stress

Ft = allow. tensile stress = 0.60 Fy
Fbx = allow. bending stress

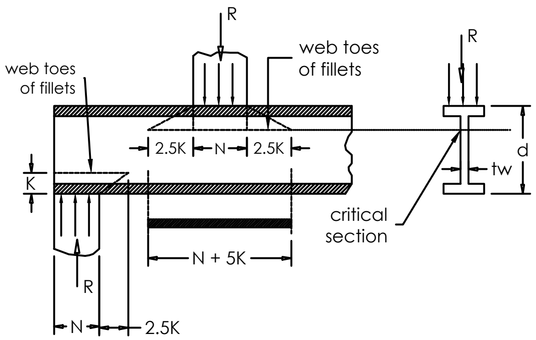
2. BENDING IN BOTH AXIS

$$\boxed{\frac{f_a}{F_t} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0}$$

where:
- F_t = allowable tensile stress
  = 0.60 Fy
- F_{bx} = 0.66 Fy (for compact section)
- F_{bx} = 0.60 Fy (for non-compact section)
- F_{by} = 0.75 Fy

LOCAL WEB YIELDING

- occurs when heavy concentrated loads produces stress at the junction of the flange and web of the beam where the load is being transferred from the relatively wide flange to the narrow web.



a. Stress at the end of the member

$$\boxed{\frac{R}{t_w (N + 2.5 K)} \le 0.66\ F_y}$$

b. Stress at the concentrated load

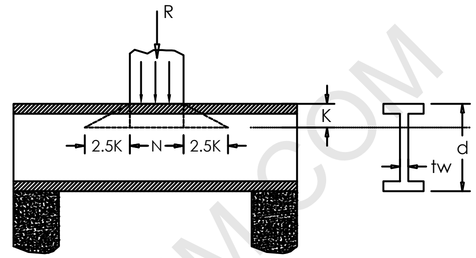
$$\boxed{\frac{R}{t_w (N + 5 K)} \le 0.66\ F_y}$$

Bearing stiffeners shall be provided if the compressive stress at the web toe of the fillets resulting from concentrated loads exceeds 0.66 Fy.

where:
- R = concentrated load or reaction in Newtons
- tw = thickness of wed in mm
- N = length of bearing (not less than K for end reactions)
- K = distance from outer face of flange to web toe of fillet in mm

WEB CRIPPING

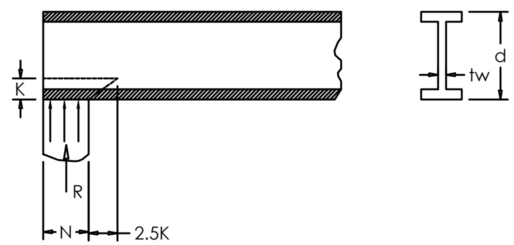
A. When the concentrated load is applied at a distance not less than d/2 from the end of the member.



$$\boxed{R = 177.2\, t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{F_{yw}\, t_f}{t_w}}}$$

where:
- F_{yw} = specified minimum yield stress of beam web in MPa

B. When the concentrated load is applied at a distance less than d/2 from the end of the member.

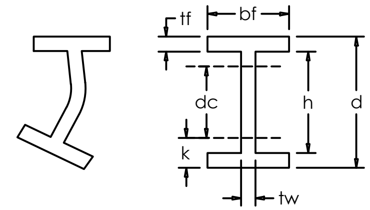


$$\boxed{R = 89.3\, t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{F_{yw}\, t_f}{t_w}}}$$

NSCP Specs: If stiffeners are provided and extend at least one half the web depth, equations A and B need not to ckeck.

SIDESWAY WEB BUCKLING

- the web will be subjected to compression if a compressive force will be applied to braced the compression flanges as a result the tension flange will buckle



A. If the loaded flange is restrained against rotation and $\frac{d_c/t_w}{L/b_f}$ is less than 2.30

$$\boxed{R = \frac{46880\, t_w^2}{h}\left[1 + 0.4\left(\frac{d_c/t_w}{L/b_f}\right)^3\right]}$$

B. If the loaded flange is not restrained against rotation and $\frac{d_c/t_w}{L/b_f}$ is less than 1.70

$$\boxed{R = \frac{46880\, t_w^2}{h}\left[0.4\left(\frac{d_c/t_w}{L/b_f}\right)^3\right]}$$

BEARING PLATES

- beams maybe supported by connections to other structural members or they may rest on concrete or masonry supports such as walls. When the support is weaker than steel, it is usually necessary to spread the load over a larger area so as not to exceed the allowable bearing stress of the weaker material.

ALLOWABLE BEARING STRESS OF CONCRETE WALL:

On full area of a concrete support

$$F_p = 0.35 f_c'$$

On less than the full area of a concrete support

$$F_p = 0.35 f_c' \sqrt{A_2/A_1} \leq 0.70 f_c'$$

A. MINIMUM WIDTH OF BEARING PLATE : (N)

1. Due to web yielding

$$\boxed{N = \frac{R}{0.66 F_y t_w} - 2.5K}$$

2. Due to web yielding

$$\boxed{R = 89.30 t_w^2 \left[1 + 3\left(\frac{N}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\frac{F_{yw} t_f}{t_w}}}$$

where:
F_{yw} = Fy if not specified

B. THICKNESS OF BEARING PLATE:

$$\boxed{t = 2n\sqrt{\frac{f_p}{F_y}}}$$

AXIAL LOAD WITH BENDING

A. DESIGN FOR AXIAL COMPRESSION AND BENDING

$$f = \frac{P}{A} \pm \frac{MC}{I} \qquad \text{(Bending in one axis only)}$$

$$f = \frac{P}{A} \pm \frac{M_x C_x}{I_x} \pm \frac{M_y C_y}{I_y} \qquad \text{(Bending in both axis)}$$

B. NSCP SPECS FOR AXIAL COMPRESSION AND BENDING

A. SMALL AXIAL COMPRESSION (fa/Fa < 0.15)

$$\boxed{\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0}$$

B. LARGE AXIAL COMPRESSION (fa/Fa > 0.15)

$$\boxed{\frac{f_a}{F_a} + \left(\frac{C_m f_b}{(1 - f_a/F_e')F_b}\right)_x + \left(\frac{C_m f_b}{(1 - f_a/F_e')F_b}\right)_y \leq 1.0}$$

Strength interaction criterion:

$$\boxed{\frac{f_a}{0.60 F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0}$$

Amplification Factor

$$\boxed{AF = \frac{1}{1 - f_a/F_e'}}$$

$$\boxed{F_e' = \frac{12\pi^2 E}{23 (KL_b/r_b)^2}}$$

Magnification Factor

$$\boxed{MF = \frac{C_m}{1 - f_a/F_e'} \geq 1.0}$$

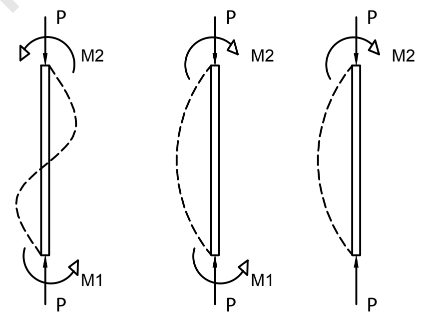
Reduction Coefficient (Modification factor)

$$\boxed{C_m = 0.60 - 0.40 (M_1/M_2)}$$

Cm = 0.85 - for members whose ends are restrained against rotation in the plane of bending

Cm = 1.0 - for members whose ends are unrestrained against rotation in the plane of bending

where:
M1 = smaller moment
M2 = bigger moment



M1 = ½ M2, Cm = 0.40 — M1/M2 is positive — Reversed Curvature

M1 = M2, Cm = 1.0; M1 = 0, Cm = 0.60 — M1/M2 is negative — Single Curvature

where:
fa = computed axial stress
fb = computed bending stress
Fa = allowable axial stress
Fb = allowable bending stress if bending moment alone existed
K = effective length factor
Lb = actual unbraced length in the plane of bending
rb = corresponding radius of gyration

ECCENTRICALLY LOADED COLUMNS USING SECANT FORMULA

Critical Column Stress

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\theta\right]$$

$$\theta = \frac{L}{2r}\sqrt{\frac{P}{EA}}$$

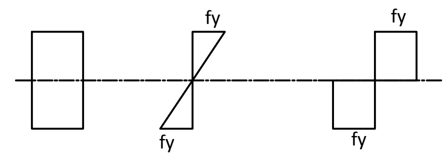
$\frac{ec}{r^2}$ = eccentricity ratio

P = total axial load

$r = \sqrt{\frac{I}{A}}$ (radius of gyration)

L = unsupported length of column

PLASTIC ANALYSIS AND DESIGN



Section — Elastic Distribution of Stress — Plastic Distribution of Stress

Plastic Neutral Axis

- the plastic neutral axis of a section is the line that divide the section into two equal areas.

Yield Moment

- moment that will just produce the yield stress in the outermost fiber of the section

$$M_y = S F_y$$

where:
S = section modulus

Plastic Moment

- moment that will produce full plasticity in a member cross section and create plastic hinge.

$$M_p = Z F_y$$

where:
Z = plastic section modulus

Shape Factor

$$\text{Shape factor} = \frac{Z}{S}$$