HISTORICAL BACKGROUND

CONCRETE

a mixture of sand, gravel, crushed rock or other aggregates held together in a rock-like mass with a paste of cement and water.

ADMIXTURES

materials added to concrete to change certain characteristics such as workability, durability and time of hardening.

JOSEPH ASPDEN

an English bricklayer who obtained a patent for Portland cement

JOSEPH MONIER

a Frenchman who invented reinforced concrete a received a patent for the const. of concrete basins and tubs and reservoirs reinforced w/ wire mesh and iron wire in 1867.

DESIGN METHODS:

1. WSD - Working Stress Design, Alternate Stress Design,or Straight-Line Design
2. USD - Ultimate Stress Design or Strength Design

PROPERTIES OF MATERIALS:

CONCRETE:

fc - allowable compressive stress of conc.
- 0.45 fc' (beams/slabs/footings)
- 0.25 fc' (columns)

fc' - specified compressive strength of conc. at 28 days curing (MPa)

$\gamma_{conc.}$ - unit weight of concrete
- 23.54 KN/m^3

Ec - modulus of elasticity of concrete
- $4700\sqrt{fc'}$ (MPa)

STEEL :

fs - allowable tensile stress of steel (MPa)

fs - 0.50 fy (beams/slabs/footings)

fs - 0.40 fy (columns)

fy - yield stress of steel (MPa)

γ_{steel} - unit weight of steel
- 77 KN/m^3

Ec - modulus of elasticity of concrete
- 200,000 MPa

TYPES OF PROBLEMS

1. Design - given the load, determine the size
2. Investigation - given the size, determine the load

MODES OF FAILURE IN BENDING

1. Crushing of Concrete - when the strain concrete reaches the ultimate strain of 0.003 mm/mm.
2. Yielding of Steel - when the actual tensile stress of steel "fs" reaches the yield stress "fy"
3. Simultaneous crushing of concrete and Yielding of Steel

TYPES OF DESIGN

1. Overreinforced - when failure is due to crushing of concrete.
2. Underreinforced - when failure is initiated by yielding of steel.
3. Balanced Design - when failure is caused by simultaneous crushing of concrete and yielding of steel

FACTORED LOAD COMBINATION

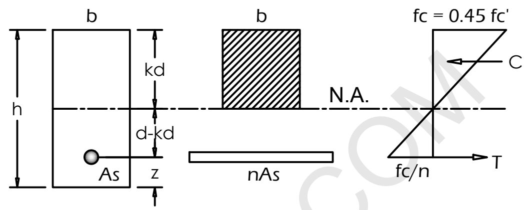
(NSCP C101-01)

1. U = 1.4DL + 1.7LL
2. U = 0.75(1.4DL + 1.7LL + 1.7 W) ; U = 0.90DL + 1.3W] ≥ (# 1)
3. U = 1.1DL + 1.3LL + 1.1E) ; U = 0.90DL + 1.1E] ≥ (# 1)
4. U = 1.4DL + 1.7LL + 1.7H ; U = 0.90DL] ≥ (# 1)

DL - Dead Load
LL - Live Load
W - Wind Load
E - Earthquake Load
H - Earth Pressure

WORKING STRESS DESIGN (WSD)

DESIGN OF BEAMS FOR FLEXURE



Beam Section   Transformed Section   Stress Diagram

where:

h = overall depth of the beam (mm)
z = steel covering (measure from the centroid of bar)
d = effective depth of the beam (mm)
d = h -z
As = area of the reinforcement (square millimeters)
fc' = compressive strength of concrete (MPa)
fs = tensile strength of steel (MPa)
b = base of the beam (mm)
n = modular ratio(always a whole number)
n = Es /Ec

Location of the neutral axis (kd)

$\Sigma M_{N.A.} = 0$
$b(kd)(kd/2) - nAs\,(d - kd) = 0$
kd = --------

Moment of Inertia of the Transformed Section

$I_{N.A.} = (1/3)(b)(kd)^3 + nAs\,(d - kd)^2$

Resisting Moment of Concrete:

Mc = C(jd)
Mc = fc/2 (b)(kd)(jd)
Mc = (1/2)(fc)(kj)(bd²)

Resisting Moment of Steel:

Ms = T jd
Ms = As fs jd

Stress of Concrete

$$fc = \frac{Mc\,(kd)}{I_{N.A.}}$$

where: Mc - resisting moment of concrete

Stress of Steel

$$\frac{fs}{n} = \frac{Ms\,(d - kd)}{I_{N.A.}}$$

where: Ms - resisting moment of steel

Compressive force of Concrete

C = 1/2 fc kd b

Tensile force of Steel

T = As fs

Moment Arm (jd)

d = jd + kd/3
j = 1 - k/3

Constant (k)

$$k = \frac{n}{n + fs/fc}$$ (For Design Only)

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$ (For Investigation Only)

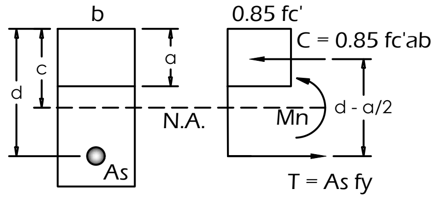
Steel Ratio

$$\rho = \frac{As}{bd}$$

ULTIMATE STRENGTH DESIGN (USD)

A. BEAMS (FLEXURAL STRESS)

1. Singly Reinforced Rectangular Beam (Reinforced in Tension Only)



Beam Section   Elevation

$a = \beta_1 c$

For fc' ≤ 30 MPa , use $\beta_1 = 0.85$

For fc' > 30 MPa ,

$\beta_1 = 0.85 - 0.008\,(fc' - 30)$

but should not be less than 0.65

$$a = \frac{As\ fy}{0.85\ fc'\ b}$$

$$\omega = \frac{\rho\ fy}{fc'}$$

$Mu = \phi\, Ru\, bd^2$ (Resisting Moment)

$Ru = fc'\omega\,(1 - 0.59\omega)$

$$\rho = \frac{0.85\ fc'}{fy}\left[1 - \sqrt{1 - \frac{2Ru}{0.85\ fc'}}\right]$$

$As = \rho bd$

Balanced Steel Ratio (ρ_b)

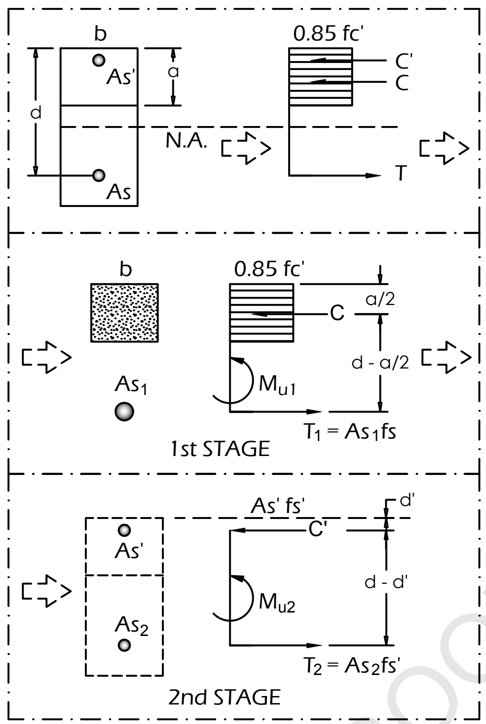
$$\rho_b = \frac{0.85\ fc'\beta_1\ 600}{fy\ (600 + fy)}$$

Maximum and Minimum Steel Ratio

$\rho_{max} = 0.75\rho_b$

$\rho_{min} = 1.4 / fy$

2. Doubly Reinforced Rectangular Beam (Reinforced in Tension/Compression)



1st STAGE	2nd STAGE
Forces:	Forces:
$C_1 = 0.85\ fc'\ ab$	$C' = As'\ fs'$
$T = As_1 fs$	$T_1 = As_2 fs\ (d-d')$
Resisting Moment:	Resisting Moment:
$M_{u1} = \phi\, 0.85fc'ab\ (d-a/2)$	$M_{u2} = \phi\, As'fs'\ (d-d')$
$M_{u1} = \phi\, As_1\, fs\ (d-a/2)$	$M_{u2} = \phi\, As_2\, fs(d-d')$

TOTAL :

$T = T_1 + T_2$ $A_S = As_1 + As_2$

$M_U = M_{U1} + M_{U2}$

where:

- a = depth of equivalent stress block
- As = area of tension reinforcement, square millimeters
- b = width of the compression face of member
- c = distance from extreme compression fiber to N.A. (mm)
- d = distance from extreme compression fiber to centroid of tension reinforcement (mm)
- d' = thickness of concrete cover measured from extreme tension fiber to center of the bar or wire, (mm)
- fc' = specified compressive stress of concrete (MPa)
- fy = specified yield strength of steel (MPa)
- Mn = nominal moment, (N-mm)
- Mu = factored moment at section, (N-mm)
- ρ = ratio of tension reinforcement = As/bd
- ρ_b = balance steel ratio
- ϕ = strength reduction factor

C. SHEAR STRESS AND DIAGONAL TENSION

$V_u = \phi V_n$

where $\phi = 0.85$

$V_n = V_c + V_s$

$V_c = 1/6\sqrt{fc'}\ bd$

Spacing of Stirrups:

$$S = \frac{Av\ fy\ d}{Vs}$$

NSCP/ACI Code Specs:

If $Vs \le 1/3\sqrt{fc'}\ bd$,Smax = d/2 or 600mm

If $Vs > 1/3\sqrt{fc'}\ bd$, Smax = d/4 or 300mm

$A_{v\,min} = bS/3fy$

V_u = factored or ultimate shear

V_c = shear force provided by conc.

V_n = nominal shear

A_{vmin} = area of steel to resist shear = 2 Asteel

FOR SINGLY REINFORCED BEAM

A. Computing Mu with given tension steel area (As)

I. Solve for $\rho = \frac{As}{bd}$

II. Check if steel yields by computing ρ_b

$$\rho_b = \frac{0.85\ fc'\beta_1 600}{fy\ (600 + fy)}$$

If $\rho \le \rho_b$ steel yields , proceed to step III

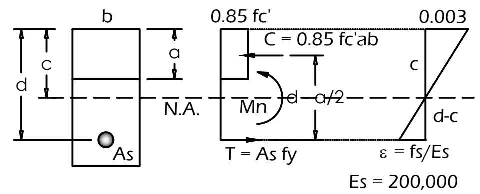
If $\rho > \rho_b$ steel does not yield , proceed to step IV

III. $\rho \le \rho_b$

$$\omega = \frac{\rho\, fy}{fc'}$$

$Mu = \phi\, fc'\, bd^2\omega\,(1 - 0.59\,\omega)$

IV. $\rho > \rho_b$ (fy = fs)



Solve for fs from the strain diagram:

$$\frac{fs/Es}{d-c} = \frac{0.003}{c}\ ;\ fs = 600\frac{d-c}{c}$$

Solve for c by summing up forces along hor.

$T = C$; $a = \beta_1 c$

$600\ As\ (d-c) = 0.85\beta_1 f'c\ b\ c^2$

Use quadratic formula to solve for "c"

Then, solve for fs and "a" with known "c"

$fs = 600\frac{d-c}{c}$; $a = \beta_1 c$

Finally, solve for Mu:

$M_u = \phi\, 0.85fc'ab\ (d-a/2)$ or

$M_u = \phi\, As\ \ fs\ (d-a/2)$

B. Computing the required tension steel area (As) of beam with given Mu

I. Solve for ρ_{max} and M_{umax}

$$\rho_{max} = 0.75\frac{0.85\,f'c\,\beta_1 600}{fy\,(600 + fy)} = \rho$$

M_{umax} = with considered factored load

$$\omega = \frac{\rho\,fy}{f'c} = \_\_\_\_\_\_$$

$$M_{umax} = \phi\,f'c\,bd^2\omega\,(1 - 0.59\,\omega)$$

If $M_u \leq M_{umax}$ design as Single Reinforced then, proceed to step II.

If $M_u > M_{umax}$ design as Doubly Reinforced

II. Solve for ρ

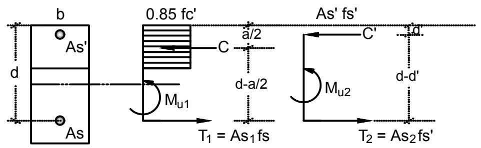
$Mu = \phi\,Ru\,bd^2$ (Solve for Ru)

$$\rho = \frac{0.85\,f'c}{fy}\left[1 - \sqrt{1 - \frac{2Ru}{0.85\,f'c}}\right] = \_\_\_\_$$

$As = \rho bd$ = ______

FOR DOUBLY REINFORCED BEAM

A. Computing As and As' of a Doubly Reinforced Beam with given Mu.



I. Solve for $As_1 = \rho_{max} bd$

II. Solve for "a" and "c": $C_1 = T_1$

$0.85\,f'c\,ab = As_1\,fy$; a = ____

$a = \beta_1 c$; c = ______

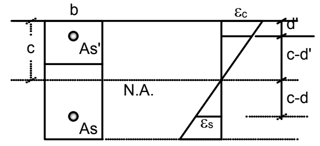
III. Solve for M_{U1}, M_{U2} and A_{s2}

$M_{u1} = \phi As_1\,fy\,(d - a/2)$

$M_{u2} = M_{U1} - M_U$

$M_{u2} = \phi As_2\,fy(d-d')$; A_{s2} = ____

IV. Verify of compression will yield.



where: $\varepsilon_{s'} = fs'/Es$

Es = 200,000

$$fs' = 600\frac{c - d'}{c}$$

If $fs' \geq fy$, proceed to step V.

If $fs' < fy$, proceed to step VI.

V. $fs' \geq fy$, then use $fs' = fy$ (compression steel yields)

$As' = As_2$

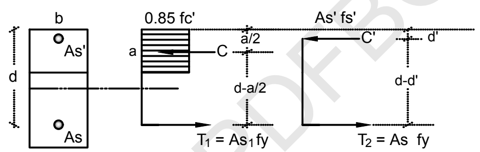
VI. $fs' < fy$, then use fs' (compression steel will not yield)

$C_2 = T_2$

$As'\,fs' = As_2 fy$

As' = ______

B. Computing Mu of a Doubly Reinforced Beam with given As and As'



I. Assume Compression steel yield

(fs' = fy)

$As_2 = As'$ = ______

$As_1 = As - As'$ = ______

II. Solve for a and c:

$[C_1 = T_1]$

$0.85\,f'c\,ab = As_1 fy$; a = ____

$a = \beta_1 c$; c = ______

III. Verify if Compression steel will yield

$$fs' = 600\frac{c - d'}{c}$$

If $fs' \geq fy$, proceed to step IV.

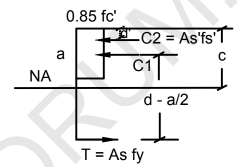
If $fs' < fy$, proceed to step V.

IV. Since $fs' \geq fy$, compression steel yields

$M_U = M_{U1} + M_{U2}$

$M_U = \phi As_1\,fy\,(d - a/2) + \phi As'\,fy(d-d')$

V. Since $fs' < fy$, assumption is wrong



$$fs' = 600\frac{c - d'}{c}$$

From stress diagram.

[C1 + C2 = T]

$0.85\,f'c\,ab + As'\,fs' = As\,fy$

$a = \beta_1 c$

$$0.85\,f'c\,\beta_1 c\,b + As'\,600\frac{c - d'}{c} = As\,fy$$

Solve for c by quadratic formula
Solve for fs' and "a"
Solve for Mu :

$$M_u = \phi\,0.85 f'c\,ab\,(d-a/2) + \phi\,As'\,fs'\,(d-d')$$

VERTICAL STIRRUP DESIGN

I. Compute the factored shear force, Vu

II. Calculate the shear strength provided by concrete, Vc

$$V_c = 1/6\sqrt{f'c}\ bd$$

If $V_u > \phi V_c$, stirrups is necessary, proceed to to Step III.

If $V_u < \phi V_c$, but $V_u > 1/2\,\phi V_c$ proceed to to Step V

If $V_u < 1/2\,\phi V_c$, stirrups are not needed

III. Calculate the shear strength Vs to be provided by the stirrup.

1. $V_n = V_u/\phi$
2. $V_s = V_n - V_c = V_u/\phi - V_c$

If $V_S \leq 2/3\sqrt{f'c}\,b_W\,d$, proceed to IV.

If $V_S > 2/3\sqrt{f'c}\,b_W\,d$, adjust the size of the beam

IV. Spacing of stirrups:

Spacing, $S = \dfrac{Av\,fy\,d}{Vs}$

If S < 25mm, increase the value of Av. either by bigger bar or shear area.

Maximum spacing, s:

If $Vs \leq 1/3\sqrt{f'c}\,bd$, Smax = d/2 or 600mm

If $Vs > 1/3\sqrt{f'c}\,bd$, Smax = d/4 or 300mm

V. If $V_u < \phi V_c$, but $V_u > 1/2\,\phi V_c$

$A_{v\,min} = bw\,S/3fy$

where S = d/2 or 600mm (whichever is smaller

TYPICAL RESISTANCE FACTORS ARE AS FOLLOWS:

SITUATION	f
Flexure, without axial load	0.90
Axial tension and axial tension w/ flexure	0.90
Shear and torsion	0.85
Compression members, spirally reinforced	0.75
Other Compression members	0.70
Bearing on concrete	0.70
Plain Concrete: flexure, compression, shear and bearing	0.65

CODE PROVISIONS: FOR DESIGN OF SINGLY-REINFORCED BEAMS

To ensure yield failure: $\rho_{max} = 0.75\rho_b$

To avoid sudden tensile failure : $\rho_{min} = \frac{0.25\sqrt{fc'}}{fy} \geq \frac{1.4}{fy}$

To control deflection: $\rho \leq 0.18\frac{fc'}{fy}$

BALANCED STEEL RATIOS

1. BEAM REINFORCED FOR TENSION

$$\rho_b = \frac{0.85\,fc'\beta_1\,600}{fy\,(600 + fy)}$$

2. BEAM REINFORCED FOR COMPRESSION

Checking Ductility

$$\bar{\rho} = \rho_b + \rho' \qquad \text{where: } \rho' = \frac{As'}{bd}$$

if $\rho < \bar{\rho}$, tension steel yields fs = fy

For compression steel

$$\rho_{lim} = 0.85\frac{\beta\,fc'\,d'\,600}{fy\,d\,(600-fy)} + \rho'$$

if $\rho < \rho_{lim}$, compression steel yields fs = fy

SHEARING STRESS OF RC BEAMS

Nominal Shear Strength Provided by Concrete:

$$V_n = V_c + V_s$$

where:

Vn = nominal shear strength of RC section
Vc = nominal shear strength provided by concrete
Vs = nominal shear strength of the shear reinforcement

For members subjected to shear and flexure only:

$$V_c = \frac{1}{6}\sqrt{fc'}\;bw\,d$$

For members subjected to axial compression:

$$V_c = \frac{1}{6}\sqrt{fc'}\;b_w\,d\left(1 + \frac{Nu}{14Ag}\right)$$

where:

Ag = gross area of section in sq.mm
Nu = factored axial load occurring with Vu
(-) for compression, (+) for tension
Nu/Ag = expressed in MPa

For members subjected to shear and flexure:

$$V_c = \frac{1}{7}\left(\sqrt{fc'} + 120\,\rho_w\,\frac{Vu\,d}{M_u}\right)b_w d$$

but shall not be greater than

$$V_c = 0.30\sqrt{fc'}\;b_w d$$

where:

$\frac{Vu\,d}{M_u} \leq 1.0$

$\rho_w = \frac{As}{b_w d}$

Mu = factored moment ocurring simultaneously w/ Vu

For members subjected to axial compression:

$$M_m = M_u - N_u\left(\frac{4h - d}{8}\right)$$

but shall not be greater than

$$V_c = 0.30\sqrt{fc'}\;b_w d\left(\sqrt{1 + \frac{0.30N_u}{A_g}}\right)$$

Substitute Mm for Mu and Vud/Mu not limited to 1.0

where, h = overall thickness of member

For members subjected to significant axial tension:

$$V_c = \frac{1}{6}\sqrt{fc'}\;b_w d\left(1 + \frac{0.30N_u}{A_g}\right)$$

where:

Nu/Ag = expressed in MPa
Nu is negative for tension
For shear reinforcement, fy $\leq$ 414 MPa.

Distance of Stirrups from support:

a. 0.50 S from face of column support
b. 0.25 S from face of beam support

T - BEAMS

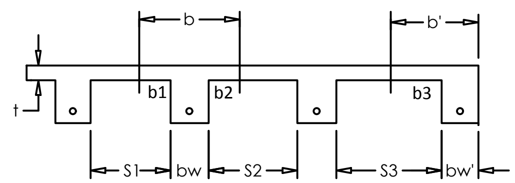
Code Requirements for T-beams

1. In T-beam construction, the flange and web shall be built integrally or otherwise effectively bonded together.
2. The width of slab effective as a T-beam shall not exceed 1/4 of the span of the beam, and effective overhanging flange on each side of the web shall not exceed:

   a) 8 times the slab thickness and
   b) 1/2 the clear distance to the next web

3. For beams with slab on one side only, the effective overhanging flange shall not exceed:

   a) 1/12 the span length of the beam,
   b) 6 times the slab thickness
   c) 1/2 the clear distance to the next web



Interior Beam End Beam

For Interior Beam

1) $b = L/4$
2) $b = 16t + b_w$
3) $b = S_1/2 + S_2/2 + b_w$

choose the smallest

For End Beam

1) $b' = L/12 + b'_w$
2) $b' = 6t + b'_w$
3) $b' = S_3/2 + b'_w$

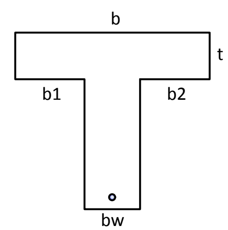
choose the smallest

For Symmetrical Interior Beam

1) $b = L/4$
2) $b = 16t + b_w$
3) b = center-center spacing of beams

choose the smallest

4. Isolated beams in which T-shape are used to provide a flange for additional compression area shall a flange thickness not less than 1/2 the width of the web and an effective flange width not more than four times the width of the web.



$t \geq b_w/2$

$b \leq 4b_w$